Lecture 14

Saturday, February 5, 2022 11:08 PM

* Prayer

* Spiritual thought

* Derivatives :

Calc I: $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

Substituting x and h by vectors becomes an issue : one can't divide by a vector. Instead, we differentiate each variable separately.

Ex $f(x,y,z) = xy + yz^2 + x^3z^2$

$f_x = y + 3x^2z^2$

$f_y = x + z^2$

$f_z = 2yz + 2x^3z$

Notation: f_x, $\dfrac{\partial f}{\partial x}$

Ex $f(x,y) = x^y$

What are f_x, f_y ?

$f = e^{y\ln x} \rightsquigarrow f_y = e^{y\ln x}\dfrac{\partial}{\partial y}(y\ln x) = e^{y\ln x}\ln x = x^y \ln x$

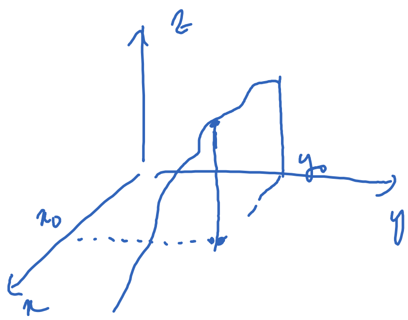
$f_x = y\, x^{y-1}$.

Higher derivatives: $f_{xx} = (f_x)_x$, $f_{xy} = (f_x)_y$,

$$f_{xyxz} = (((f_x)_y)_x)_z, \ldots$$

Geometric meaning

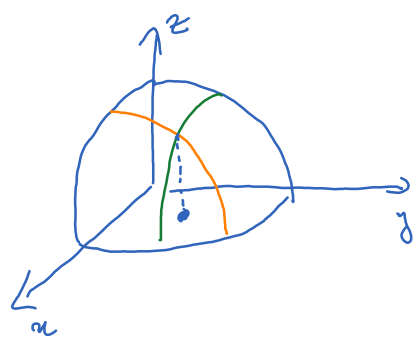
$$f_x(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$



$f_x(x_0,y_0)$ = rate of change along the curve

$$\begin{cases} x = x \\ y = y_0 \\ z = f(x, y_0) \end{cases}$$

Ex



$$f(x,y) = \sqrt{1 - x^2 - y^2}$$

Clairaut's theorem:

If f is differentiable, f_x & f_y are continuous then